A SOLID–LIQUID MODEL OF AN EXPLOSION IN THE GROUND

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The basic idea proposed in [1] of the solid–liquid model of an explosion in the ground consists in that in regions close to the explosion the tensile forces are small in comparison with the pressure and the inertial forces, and the medium here can be assumed to be ideal. High velocities and displacements (liquid zone) are characteristic for this region. Remote from the explosion, where just tensile forces predominate, the velocities and displacements are small, and the medium is almost motionless (seismic zone). These zones are separated by a transition layer. As a first approximation, it is assumed that this layer is infinitely thin and is a sealed boundary for the moving ideal medium. If the compressibility of the ground is neglected, then for the incompressible ideal medium in the liquid zone of the explosion a pulsed formulation is applicable, in which the action of the explosion is described in terms of the pressure pulse exerted on the medium by the explosion products.

The boundary of the liquid zone in this case is found from the solution of the impulse problem, if a certain supplementary condition of a physical nature is specified at this boundary.

In [2] it was proposed to assume for this condition that the velocity of motion v along the boundary is equal to some critical value

$$v = c. \tag{1}$$

With this condition effective solutions are obtained which enable the shape of the blast crater to be determined in a large number of problems, with different geometry of charge placement (for example, in [3, 7]).

Comparison of the solutions found for the example of a cord-charge explosion with the experimental data showed [8] that the theoretical relation obtained between the width of the crater and the depth of burial and size of the charge, with an appropriate choice of c can describe the experimental results. Concerning the shape of the craters, the congruity with experiment is more qualitative.

In order to refine this question, a series of experiments with a surface-laid cord charge was carried out. The theoretical shape of the crater obtained in the ground explosion model described above is shown in Fig. 1. The solution of this problem is characteristic in that the shape of the crater (in contrast to its size) is independent of the charge size or of the magnitude of c. Therefore, verification of the model by means of a comparison of this solution with the experimental data is particularly simple.

Glass tubes (1.2 m long, 7 mm diameter, and filled with hexogen) were used as the charges and were implanted in the ground at a depth equal to its radius (flush with the surface). After each explosion the ground was sliced with a thin piece of sheet metal, on which millimeter graph paper was glued, and then the contour of the crater was traced on the graph paper. The line of the free surface before the explosion was determined from previously inscribed reference marks.

In Fig. 2a–c, the curves 1 show the profiles of the craters obtained, respectively, in wet consolidated sand (density $\rho = 1.74\ g/cm^3$, moisture content $w = 13.7\%$), in water-saturated sand ($\rho = 1.86\ g/cm^3$, $w = 20\%$), and water-saturated silty ground ($\rho = 1.9\ g/cm^3$, $w = 17\%$). The half-width of the crater is chosen as the scaling unit for each profile. The true width of the crater has dimensions of 34, 34, and 50 cm, respectively. It can be seen that the shape of the craters depends significantly on the properties of the soil (water saturability, plasticity). This result of the experiments leads to the necessity for improving the model of the phenomenon.

Let us consider the state of the ground at the crater edge at the instant when the pressure pulse from the detonation products is transmitted almost entirely to the ground. In this

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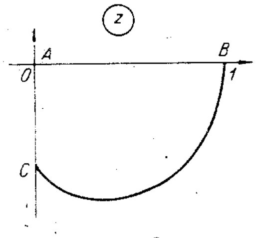

Fig. 1

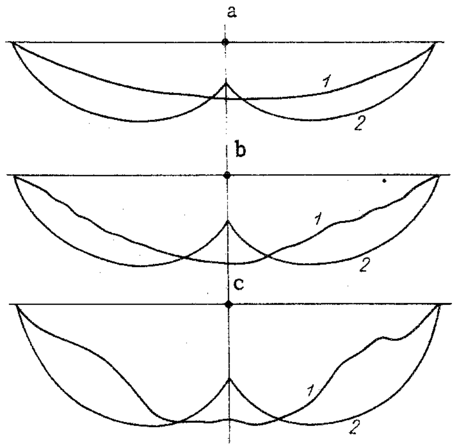

Fig. 2

case, the velocity of the ground near the boundary is v, the pressure is p ($p = -\varphi\rho/\tau$, where φ is the flow potential at this position, τ is the time of application of the pressure pulse, and ρ is the density). As the critical condition at the crater boundary, we shall assume that the shear forces exerted from the direction of the moving ground on the motionless ground are equal to the maximum shear forces which can be endured by the motionless ground — its ultimate shear strength.

It is well known [9] that the limiting maximum shear stresses τ_1 can be approximated by the Coulomb–Mohr law over a wide range of grounds.

In the case being considered it is convenient to use its approximate expression $\tau_1 = a_1 + b_1 p$, where p is the pressure. In soil dynamics [10] there is a similar linear dependence between the intensity of the tangential stresses T_2 and the pressure p. As the intensity of the tangential stresses is equal approximately to their maximum value τ_2 ($T_2 \approx 1.08\tau_2 \pm 7\%$ [11]), we shall assume that the magnitude of the tangential stresses at the crater boundary, created by the dry friction of the moving ground over the motionless ground, is determined approximately by the expression $\tau_2 = a_2 + b_2 p$. In addition to the forces of dry friction, during motion in the ground there are forces of viscous friction. The various methods for introducing viscosity into the equations of soil dynamics can be found in [12-14].

Assuming that the shear stresses τ_3 due to viscosity are proportional to the velocity gradient, at the crater boundary we have

$$\tau_3 \approx \mu \partial v/\partial n \approx \mu v/h,$$

where n is the normal to the boundary; h is the thickness of the transitional layer; and μ is the coefficient of viscosity.

As the critical condition at the crater boundary, we shall assume

$$\tau_1 = \tau_2 + \tau_3.$$

Substituting p by its value $-\varphi\rho/\tau$ and assuming that τ and h for a given explosion are constant, we obtain a linear relation between the velocity and the flow potential,

$$v = c + k\varphi. \tag{2}$$

This condition for $k = 0$ coincides with the previous assumption (1).

This ratio of the terms in Eq. (2) depends on the properties of the ground. Obviously, the second term is less important for media with low stability (metals, ground with plastic, and clayey admixtures). The water saturation of the ground also should lead to a reduction of the effect of this term.

On the other hand, for dry sand, for which the effect of strengthening (compaction) by the action of the pressure is large, the magnitude of the coefficient k also may be large.

Let us consider the similar problem of the explosion of a surface cord charge in the solid–liquid model with the supplementary condition (2) at the crater boundary. Figure 1 shows a section in the plane perpendicular to the charge. In view of the symmetry of the problem, we shall confine ourselves to the right-hand half-plane.

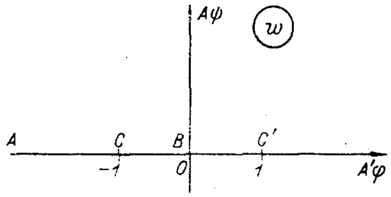

Fig. 3

We introduce the complex flow potential

$$w(z) = \varphi(x,\ y) + i\psi(x,\ y)$$

and we shall formulate the problem in the following way. It is required to find in the region ABC an analytic function w(z) and the unknown part of the boundary CB according to the following boundary conditions: at the free surface AB, $\varphi = 0$; on the line of flow ABC, $\psi = 0$; and at the unknown boundary, $|dw/dz| = c(1 - b\varphi)$.

Here $b = \text{const} \geq 0$ is the additional parameter of the problem. We denote the value of the potential at the point C by α, and we introduce the dimensionless variables

$$\bar{w} = w/\alpha,\ \bar{z} = zc/\alpha,\ \bar{b} = b\alpha$$

(in future, we shall omit the dashes for simplicity). Let us consider the function

$$\omega(w) = \ln\,(idz/dw).$$

In the plane w, the region of flow ABC is the second quadrant with the conformity of the points shown in Fig. 3. As $\operatorname{Im}\omega = \pi$ on AB, then by the principle of symmetry [15], the function ω can be extended on the whole upper half-plane $\operatorname{Im} w > 0$ (this will be continued in the plane z through the free surface). Thus, the boundary problem results when it is required to find the analytic function $\omega(w)$ in the upper half-plane, assuming the following values on the real axis:

$$\operatorname{Im}\omega = 0 \quad \text{for} \quad -\infty < \varphi < -1,$$
$$\operatorname{Re}\omega = -\ln\,(1 + b|\varphi|) \quad \text{for} \quad |\varphi| < 1,$$
$$\operatorname{Im}\omega = 2\pi \quad \text{for} \quad 1 < \varphi < \infty.$$

Let us consider the function

$$\omega_1 = -2\,ln\,[e^{-\pi i}(w + \sqrt{w^2 - 1})].$$

On the real axis, it assumes the following values:

$$\operatorname{Im}\omega_1 = 0 \quad \text{for} \quad -\infty < \varphi < 1,$$
$$\operatorname{Re}\omega_1 = 0 \quad \text{for} \quad |\varphi| < 1,$$
$$\operatorname{Im}\omega_1 = 2\pi \quad \text{for} \quad 1 < \varphi < \infty.$$

Then for the function $\omega_2 = \omega - \omega_1$ we obtain the Keldysh–Sedov boundary problem [15]

$$\operatorname{Im}\omega_2 = 0 \quad \text{for} \quad |\varphi| > 1,\ \psi = 0,$$
$$\operatorname{Re}\omega_2 = -\ln\ (1 + b\,|\varphi|) \quad \text{for} \quad |\varphi| < 1,\ \psi = 0,$$

the solution of which is the formula

$$\omega_2 = \frac{\sqrt{w^2 - 1}}{\pi}\left[\int_{-1}^{0} \frac{\ln\,(1 - b\tau)\,d\tau}{(\tau - w)\sqrt{1 - \tau^2}} + \int_{0}^{1} \frac{\ln\,(1 + b\tau)\,d\tau}{(\tau - w)\sqrt{1 - \tau^2}}\right]. \tag{3}$$

As $\omega_1 + \omega_2 = \omega \equiv \ln(idz/dw)$, for z(w) we obtain the differential equation

$$idz/dw = e^{-\omega_2}\,(w + \sqrt{w^2 - 1})^{-2}.$$

The solution of this equation in the range $0 \geq \varphi \geq -1$, where $\psi = 0$, allows the shape of the crater to be found. The integrations were carried out numerically. For this (by means of Sokhotskii's formula for the boundary value of an integral of the Cauchy type [15]), expression (3) for ω_2 at the boundary was reduced to a form suitable for evaluation:

$$\omega_2 = -\ln(1-b\varphi) + \frac{i\sqrt{1-\varphi^2}}{\pi}\left\{\int_0^1 \frac{\ln(1+b\tau)\,d\tau}{(\tau-\varphi)\sqrt{1-\tau^2}} + \int_{-1}^{0}\left[\frac{\ln(1-b\tau)}{\sqrt{1-\tau}} - \frac{\ln(1-b\varphi)}{\sqrt{1-\varphi}}\right]\frac{d\tau}{(\tau-\varphi)\sqrt{1+\tau}}\right\} + \frac{i}{\pi}\ln(1-b\varphi)\ln\frac{1-\sqrt{1-\varphi}}{1+\sqrt{1-\varphi}}.$$

The calculations showed that with increase of the parameter b the craters become relatively shallower. Curves 2 in Fig. 2a–c show the shape of the craters obtained for values of the parameter b = 100, 20, and 3, respectively. These values are chosen from the condition of compatibility of the theoretical crater shapes with the experimental values.

It can be seen from Fig. 2 that this theory enables the experimental shapes of the craters obtained in different soils by the explosion of a surface cord charge to be described satisfactorily.

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